Dephasing from interactions and spin disorder

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We calculate the dephasing rate of the electrons in the presence of interactions and elastic spin disorder scattering. In the frame of a self-consistent diagrammatic treatment, we obtain saturation of the dephasing rate in the limit of zero temperature for spin-orbit disorder in 2 dimensions. This result is in agreement with relevant experiments.

The dephasing rate τ_ϕ^{-1} provides a measure of the loss of coherence of the carriers. The saturation of the dephasing rate at low temperature seen in *some* samples[1−9] has attracted a vigorous interest.

In the presence of spin-less disorder, the Cooperon (particle-particle diffusion correlator) is given by

$$C_0(q,\omega) = \frac{1}{2\pi N_F \tau^2} \frac{1}{Dq^2 - i\omega + \tau_\phi^{-1}} \quad . \tag{1}$$

D is the diffusion coefficient, N_F is the density of states at the Fermi level and τ^{-1} the total impurity scattering rate. We will work in the diffusive regime $\epsilon_F \tau > 1$ ($\hbar = 1$), ϵ_F being the Fermi energy.

Previous studies[10−20] have focused on the calculation of τ_ϕ^{-1} in the absence of spin-scattering disorder. Here we calculate τ_ϕ^{-1} in the presence of spin-scattering disorder, whereupon the Cooperon becomes spin-dependent. The relevant terms C_i are shown in fig. 1. We start by giving the explicit form of these $C^o_{1,2}$ *without* a dephasing rate. For the case $\tau_{so}^{-1} > 0, \tau_S^{-1} = 0$ - with τ_{so}^{-1} the spin-orbit impurity scattering rate and τ_S^{-1} the magnetic impurity scattering rate - they are given by[21]

$$C^o_{1,2} = b_{1,2} \Big[\frac{1}{Dq^2 - i\omega} - \frac{1}{Dq^2 - i\omega + 3/(4\tau_{so})} \Big] \quad , \tag{2}$$

with $b_1 = (3\tau_{so}/\tau - 2)/(2u), b_2 = -1/u, u = 4\pi N_F \tau^2$. We should emphasize that the impurity scattering considered is elastic.

To calculate the dephasing rate, we solve the appropriate Bethe-Salpeter equations for *all* three Cooperons $C_i(q,\omega)$, $i = 0,1,2$. These are shown schematically in fig. 2:

$$C_0 = C^o_0 + C^o_0 W C_0 \quad , \tag{3}$$

$$C_1 = C^o_1 + C^o_1[FC_1 + RC_2] + C^o_2[RC_1 + FC_2] \quad , \tag{4}$$

$$C_2 = C^o_2 + C^o_2[FC_1 + RC_2] + C^o_1[RC_1 + FC_2] \quad . \tag{5}$$

In fig. 3 we show explicitly the components of the self-energy terms $\Sigma = W, F, R$. These quantities are given by

$$W = \Sigma_0 + d_0 \Sigma_1 \quad , F = \Sigma_1 + d_1 \Sigma_0 \quad , R = d_2 \Sigma_2 \quad . \tag{6}$$

Here, $d_0 = d_1 = \{1/(\epsilon_F\tau) + 4\pi\}/(2\pi^2 \epsilon_F \tau_{so})$ and $d_2 = -2\{1/(\epsilon_F \tau_{so}) + 2\pi\tau/\tau_{so}\}/\pi$. The terms containing the factors d_i - with a spin impurity line either looping around the Cooperon or crossing it - provide the coupling between the spin-independent and the spin-dependent Cooperons, and they turn out to be *crucial* for the saturation of the dephasing rate obtained below. In the spirit of ref.[11] we obtain

$$\Sigma_i \equiv \Sigma_i(q=0,\omega=0) \simeq -\sum_q \int_{-\infty}^{\infty} dx \frac{C_i(q, x+i0)\, \mathrm{Im} V(q, x+i0)}{\sinh(x/T)} \quad , \tag{7}$$

where

$$V(q,\omega) = \frac{v_q}{1 + v_q \Pi(q,\omega)} \quad , \; \Pi(q,\omega) = \frac{N_F}{2} \Big[\frac{Dq^2}{Dq^2 - i\omega} + \frac{Dq^2 + \tau_{sp}^{-1}}{Dq^2 - i\omega + \tau_{sp}^{-1}} \Big] \quad . \tag{8}$$

Here $v_q = 2\pi e^2/q$ is the bare Coulomb interaction and $\tau_{sp}^{-1} = (4/3)(\tau_S^{-1} + \tau_{so}^{-1})$ the total spin scattering rate. We make the approximation[11] $\int_{-\infty}^{\infty} dx F(x)/\sinh(x/T) \simeq T \int_{-T}^{T} dx F(x)/x$. The bosonic modes with energy greater than T manifestly do *not* contribute to the self-energy and the dephasing process.

Since the coefficients $d_i \ll 1$ for $\epsilon_F \tau_{so} \gg 1$, we can decouple a 2×2 system of equations involving only $C_{1,2}$, to facilitate the solution of eqs. (3-5). Subsequently, $C_{1,2}$ are given by

$$C_{1,2}(q,\omega) = S_{1,2}\Big[\frac{1}{Dq^2 - i\omega + r_-} - \frac{1}{Dq^2 - i\omega + r_+}\Big] \quad . \tag{9}$$

Here $r_+ = (b_1 - b_2)(\Sigma_2 - \Sigma_1), r_- = -(b_1 + b_2)(\Sigma_2 + \Sigma_1)$, $S_i = a_o\{b_i(Dq^2 - i\omega) + M_i\}/X, X = 2(b_1\Sigma_2 + b_2\Sigma_1)$ and $a_o = 8N_F\tau^4/D$.

The self-energies are given by

$$\Sigma_i = \frac{Ta_o}{X}\Big\{(M_i - b_i r_-)\ln\Big(\frac{z_o - r_-}{T - r_-}\Big) - (M_i - b_i r_+)\ln\Big(\frac{z_o - r_+}{T - r_+}\Big)\Big\} \quad , \tag{10}$$

with $i = 1, 2$, $z_o = \tau^{-1}$ and $M_1 = (b_2^2 - b_1^2)\Sigma_1, M_2 = -(b_2^2 - b_1^2)\Sigma_2$.

Taking

$$b_1 = -b_2 \quad (\tau_{so} = \frac{4}{3}\tau) \quad , \tag{11}$$

we obtain a simplified version of these equations:

$$\Sigma_1 = Ta_o b_1 \ln\Big(\frac{z_o + 4b_1\Sigma_1}{T + 4b_1\Sigma_1}\Big) \quad , \tag{12}$$

and $\Sigma_2 = -\Sigma_1$. In the limit $T \to 0$ we obtain the solution

$$\Sigma_1 = -z_o/(4b_1) + O(e^{-1/T}) \quad . \tag{13}$$

This finite solution for $\Sigma_{1,2}$ for $T \to 0$ is a generic fact for any finite value of τ_{so}^{-1}.[22]

In 1-D we obtain similarly

$$\Sigma_i = \frac{Ta_{1D}}{X}\Big\{\frac{M_i - b_i r_-}{\sqrt{r_-}}\Big[f(z_o, r_-) - f(T, r_-)\Big] - \frac{M_i - b_i r_+}{\sqrt{r_+}}\Big[f(z_o, r_+) - f(T, r_+)\Big]\Big\} \quad , \tag{14}$$

with $f(x,y) = \ln\{(\sqrt{x} - \sqrt{y})/(\sqrt{x} + \sqrt{y})\}$ and $a_{1D} = 2\pi\sqrt{D}a_o$. In 3-D we have

$$\Sigma_i = \frac{Ta_{3D}}{X}\Big\{b_i X\sqrt{z_o} + (M_i - b_i r_-)\sqrt{r_-}\Big[f(z_o, r_-) - f(T, r_-)\Big] - (M_i - b_i r_+)\sqrt{r_+}\Big[f(z_o, r_+) - f(T, r_+)\Big]\Big\} \quad ,$$

with $a_{3D} = 2a_o/\pi$. Both the 1-D and 3-D cases differ from 2-D in that no *finite* solution exists for $\Sigma_{1,2}$ in the limit $T \to 0$. This is due to the presence of the factors $\sqrt{r_\pm} \propto \sqrt{\Sigma_i}$ in the right-hand side of the respective eqs. for Σ_i.

It turns out that

$$\tau_\phi^{-1} \simeq -b_0 d_0 \Sigma_1 = \frac{2[4\pi + 1/(\epsilon_F\tau)]}{\pi^2\tau(\epsilon_F\tau_{so})(3\tau_{so}/\tau - 2)} \quad , \tag{15}$$

with $b_0 = 1/(2\pi N_F\tau^2)$. Hence τ_ϕ^{-1} *saturates* in the limit $T \to 0$ in 2-D. This is to be contrasted with the absence of spin disorder, where it has been shown that $\tau_\phi^{-1} \propto T\ln T \to 0$[10–15], and this fact yields eq. (15). Moreover, we would like to emphasize that other processes in Σ, which are first order in the interaction $V(q,\omega)$, e.g. with V crossing diagonally the Cooperon, do not modify qualitatively the result in eq. (15).

Now, the total correction to the conductivity can be written as

$$\delta\sigma_{tot} = \delta\sigma_o + \delta\sigma_{sp} \quad , \tag{16}$$

where the first term is the spin independent one - involving C_0 - and the second term the spin dependent one - involving $C_{1,2}$. The latter is expected to saturate always in the low T limit. The form usually fit to experiments[1,3] for the former is

$$\delta\sigma_o = \sigma_0\Big[\ln(y) - \Psi(y + \frac{1}{2})\Big] \quad , \tag{17}$$

where Ψ is the digamma function, $y = 1/4eHD\tau_\phi$, H the magnetic field and $\sigma_0 = e^2/(2\pi^2)$. Obviously, this expression saturates for a saturating τ_ϕ^{-1}, and vice-versa for the experimental determination of τ_ϕ^{-1}.

A number of experiments show saturation of τ_ϕ^{-1} in the zero temperature limit. The samples in which saturation is observed are made of elements with a high atomic number, which induces a substantial spin-orbit scattering. Moreover, several of the samples, in which the dephasing saturation is observed, are truly 2-dimensional, such as the wires in refs.[1–3], the quantum dots in refs.[4,5] etc. We believe that the observed saturation can be understood in the frame of our results above.

I have enjoyed useful discussions/correspondence with Peter Kopietz and Pritiraj Mohanty.

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[22] Note that $\Sigma_1 \propto T \ln{(z_o/T)} \to 0$ is also a solution, yielding however $C_{1,2}(q,\omega) \to 0$ identically.

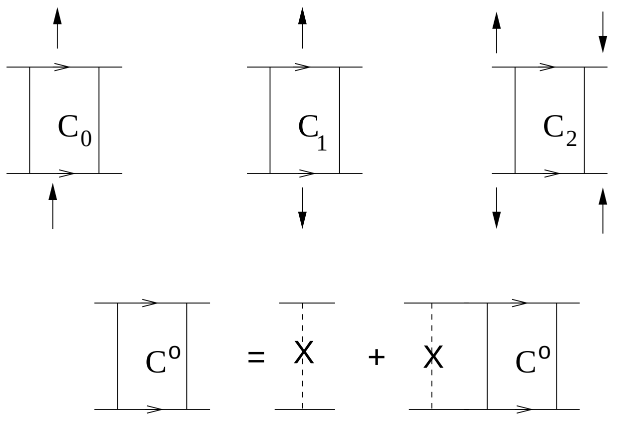


Fig. 1

The 3 Cooperons C_0, C_1, C_2. Note the spin indices. The Cooperons C_i^0 do not contain a dephasing rate. The dashed line with the cross stands for impurity (disorder) scattering.

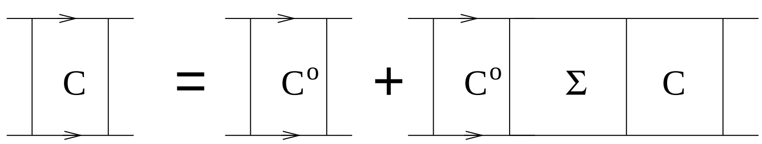
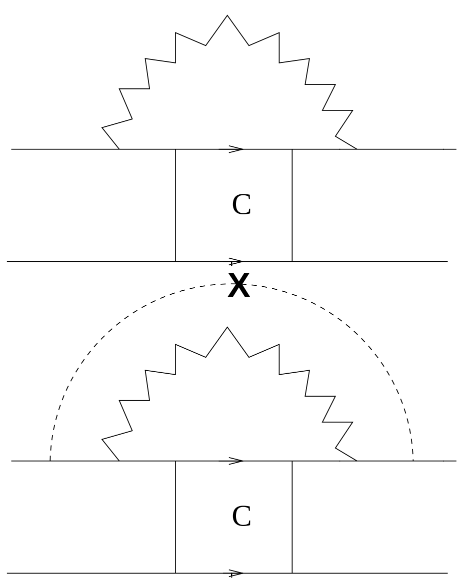


Fig. 2

Schematic form of the equations (3-5) involving the Cooperons and the self-energies Σ.



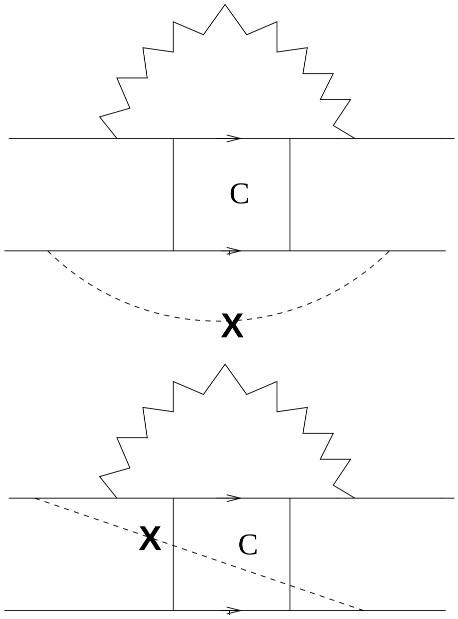


Fig. 3

The various components of the self-energy. The wiggly line represents the screened Coulomb interaction of eq. (8). The diagrams with the extra spin-disorder impurity line are crucial for the saturation of $\tau_\phi^{-1}(T \to 0)$. We consider *all* possible variations of the diagrams shown here.